1899MNRAS..59..524R

A further Investigation concerning the Position Error affecting Eye-estimates of Star Magnitudes. By Alexr. W. Roberts.

In No. 6, vol. lvii. of the *Monthly Notices*, I entered upon an investigation dealing with the amount and nature of the position error affecting visual determinations of star magnitudes. The results obtained in that article were based on an examination of the differences between the direct and reverse magnitude determinations of a group of stars surrounding the short period variable Lac. 5861. Since then it has seemed to me an important addition to the investigation to consider only naked-eye estimates of star magnitudes.

Results based on such estimates would be entirely free from the suspicion that the error might possibly be due to instrumental causes. Further, a comparison could be instituted between the results obtained from naked-eye observations (observations made with both eyes) and the results obtained from telescopic observations (one eye only being used); and this comparison would indicate if the amount and character of the variation were the same whether the right or left eye was used in making the observations.

Several groups of stars were available for the proposed investigation; the same conclusion, however, can be arrived at, and more directly, by considering only two stars, instead of several stars, of a group.

North and south of the well known short period variable κ *Pavonis* are two stars of almost equal magnitude, which have been regularly used along with others in the near neighbourhood as comparison stars for κ *Pavonis.*

The two stars and their positions are

	h m s	° ′ ″
ζ Pavonis	18 28 25	−71 31 50 (1875)
λ Pavonis	18 40 38	−62 19 38

It was the constantly changing magnitudes of these two stars, as observations in varying hour angles were made, that first impressed me with the reality of the phenomenon of position error, and the importance of a thorough investigation as to its nature.

As the hour angle of the stars varies, it is evident that their relative position will also vary. Sub-polo, λ *Pavonis* is the lower of the two stars, and it then seems at least half a magnitude brighter than ζ *Pavonis.*

As the stars rise higher and higher this difference in brightness diminishes, until at an altitude of 50° the two stars seem equal in magnitude. As the stars rise to their upper culmination ζ *Pavonis* becomes the brighter, reaching its maximum brightness relative to λ soon after passing the meridian. It then decreases in brightness as its distance below λ *Pavonis* diminishes,

1899MNRAS..59..524R

becoming soon after passing its lower culmination, as already said, half a magnitude fainter than λ *Pavonis*.

This states generally the relation of the variation in the brightness of the two stars to their varying relative positions. We are able, however, as in the case of the stars surrounding Lac. 5861, to deal more rigorously with the matter.

It is evident that as ζ *Pavonis* and λ *Pavonis* circle round the heavens they take up relatively the same position to one another as they would if they revolved round a point midway between the stars. Practically, therefore, for the question to be considered, we may regard ζ and λ *Pavonis* as revolving round this point instead of moving round the heavens.

The co-ordinates of this middle point, which we may call O, are

$$\text{R.A. } 18^{\text{h}}\ 35^{\text{m}}\ 40^{\text{s}}\ (1875)$$

$$\text{Dec. } -66°\ 56'\ 9''$$

and the angle formed between the line joining the two stars and the line passing through O and the pole is

$$7°\ 7'\ 26''.$$

Let m_ζ be the true magnitude of ζ *Pavonis*, then this magnitude will be affected by

(1) *Atmospheric absorption*. The amount of loss due to this cause can be expressed by the quantity k sec z, k being the co-efficient of absorption and z the zenith distance of the star.

(2) *Relative position with regard to O*. It would appear that the simple trigonometrical expression

$$p \cos(\theta - \text{M}),$$

where θ is the parallactic angle of O, satisfies the variation in magnitude due to this cause. A second term

$$q \cos(2\theta - \text{N})$$

was added in the first discussion of the observations, but it was found that the number of unknown quantities then introduced would overburden the equations.

Thus at any instant of time the observed magnitude of ζ *Pavonis* will be

$$m_\zeta + k \sec z_\zeta + p \cos(\theta - \text{M}).$$

Similarly the observed magnitude of λ *Pavonis*, at the same instant, will be

$$m_\lambda + k \sec z_\lambda - p \cos(\theta - \text{M});$$

and the *difference* between the observed magnitudes of the two stars,

$$(m_\zeta - m_\lambda) + k(\sec z_\zeta - \sec z_\lambda) + 2p \cos(\theta + \text{M}).$$

1899MNRAS..59..524R

As the difference between the magnitudes may vary continually during the period over which the observations extend, to the above expression a term

$$m' \, (t - 1895)$$

should be added.

Putting,

$$m_\zeta - m_\lambda = x$$
$$\text{Sec } z_\zeta - \sec z_\lambda = s$$
$$2p \cos \text{M} = a$$
$$2p \sin \text{M} = \beta$$

the type of equation of condition connecting the observed magnitudes of ζ and λ *Pavonis* becomes

$$\text{obs. mag. of } \zeta - \text{obs. mag. of } \lambda = x + m' \, (t - 1895) + sk + a \cos \theta + \beta \sin \theta.$$

In the following table are given all the data necessary for the determination of the unknown quantities

$$x, \; m', \; k, \; a, \text{ and } \beta$$

It may be stated that the differences between the observed magnitudes of ζ and λ are not obtained by direct comparison when the amount of difference is as great as $0^{\text{m}}\cdot5$, or even less. In this case intermediate magnitudes are used as sequences.

Rot. No.	No. of Obsns.	$(t - 1895)$	Secz_ζ − Secz_λ	Hour Angle of ☉.	Par. Angle of ☉ = θ.	$\zeta - \lambda$.	Weight.	Computed $\zeta - \lambda$.
				h m	°	m		m
1	2	+0·3	−3·73	13 54	24·2	+0.40	1	+0·37
2	5	0·9	2·05	14 58	37·7	+0·50	2	+0·51
3	17	0·4	1·13	15 55	49·4	+0·48	3	+0·53
4	25	0·6	0·53	16 53	61·6	+0·48	4	+0·51
5	24	0·7	0·22	17 55	74·8	+0·49	4	+0·43
6	30	0·4	−0·04	18 57	88·5	+0·40	4	+0·29
7	23	0·3	+0·04	19 56	102·5	+0·23	4	+0·13
8	12	0·1	0·08	20 58	118·9	−0·23	3	−0·08
9	11	0·3	0·11	21 56	136·1	−0·48	3	−0·30
10	15	1·5	0·13	22 50	154·3	−0·65	3	−0·48
11	13	1·3	0·14	23 56	178·5	−0·66	3	−0·70
12	10	0·9	0·15	0 55	200·4	−0·71	3	−0·85
13	8	0·6	0·15	1 53	220·4	−0·75	3	−0·86
14	5	1·0	0·13	2 52	238·4	−0·82	2	−0·85
15	4	1·1	0·10	4 0	256·5	−0·72	2	−0·70
16	6	+0·9	+0·06	4 47	267·8	−0·65	2	−0·55
17	3	−0·1	−0·06	5 58	283·7	−0·50	1	−0·40

1899MNRAS..59..524R

The resulting normal equations are—

$$\begin{array}{rrrrrl}
+47{\cdot}00x & +31{\cdot}30m' & -11{\cdot}42k & +\ 9{\cdot}09\alpha & -15{\cdot}74\beta & = -\overset{\mathrm{m}}{6{\cdot}08} \\
+31{\cdot}30 & +28{\cdot}24 & -\ 5{\cdot}53 & +10{\cdot}12 & -\ 5{\cdot}08 & \quad -\ 8{\cdot}57 \\
-11{\cdot}42 & -\ 5{\cdot}53 & +27{\cdot}83 & +12{\cdot}23 & +\ 9{\cdot}59 & \quad -\ 8{\cdot}43 \\
+\ 9{\cdot}09 & +10{\cdot}12 & +12{\cdot}23 & +17{\cdot}51 & +\ 4{\cdot}36 & \quad -13{\cdot}30 \\
-15{\cdot}74 & -\ 5{\cdot}08 & +\ 9{\cdot}59 & +\ 4{\cdot}36 & +29{\cdot}48 & \quad -12{\cdot}18
\end{array}$$

which being solved gives

$$\begin{array}{rl}
 & \mathrm{m} \\
x = & -0{\cdot}14 \\
m' = & +0{\cdot}02 \\
k = & +0{\cdot}07 \\
\alpha = & -0{\cdot}65 \\
\beta = & -0{\cdot}41
\end{array}$$

and

$$\begin{array}{rl}
M = & 212^{\circ}\ 30' \text{ or } 32^{\circ}\ 30' \\
2p = & +0^{\mathrm{m}}{\cdot}77.
\end{array}$$

Therefore,

Mag. of ζ Pavonis − Mag. of λ Pavonis

$$= 0^{\mathrm{m}}{\cdot}14 + 0^{\mathrm{m}}{\cdot}02\ (t-1895) + 0^{\mathrm{m}}{\cdot}07(\sec z_{\zeta} - \sec z_{\lambda}) + 0^{\mathrm{m}}{\cdot}77 \cos(\theta - 32^{\circ}\ 30').$$

The values computed from this final equation are given in the last column of the preceding table. It may be objected that the residuals are large : this may be admitted, but that they are so arises, no doubt, from the impossibility of expressing the varying sensitiveness of different portions of the retina in an exact mathematical form.

We may consider now the several results obtained.

(1) *Magnitude of ζ and λ Pavonis.* The magnitude of *ζ Pavonis*, as obtained from observations, is $4^{\mathrm{m}}{\cdot}30$: the value of λ *Pavonis* consequently is $4^{\mathrm{m}}{\cdot}30 - x = 4{\cdot}44$. The U.A. values are 4·20 and 4·30 respectively. The small value of m', $0^{\mathrm{m}}{\cdot}02$, a quantity less than its weight, indicates still further the permanence of the light of both stars.

(2) *Co-efficient of atmospheric absorption.* The value obtained in the present investigation for this important coefficient is

$$+0^{\mathrm{m}}{\cdot}07.$$

It is possible that this value is slightly too small. I am convinced, however, that the value obtained at Harvard from observations made in the years 1886, 1887, and 1888, and the value generally accepted, is too great, at least for these latitudes of dry atmosphere and clear skies.

1899MNRAS..59..524R

The Harvard value is

$$+0^{m}\cdot 39.^{*}$$

This would mean that a first magnitude star could not be seen on the horizon. Every clear night at this season I can roughly time my watch by the setting of *Sirius*. As the star reaches the horizon, a straight black line against the dark blue beyond, it flickers for a few seconds, then disappears. Sixth magnitude stars have also been discerned at an altitude of 5°.

From considerations such as these, as well as from the definite value obtained in this paper, it is certain that the value of the coefficient of absorption cannot be greater than one-tenth of a magnitude for places with the same atmospheric conditions as Lovedale.

(3) *Amount and character of Position Error.* The expression

$$0^{m}\cdot 77 \cos(\theta - 32^{\circ}\ 30')$$

indicates that the maximum variation due to relative position is reached when the parallactic angle of the centre O is 32° 30′. Now the line joining ζ and λ makes an angle of 7° 7′ with the meridian line: thus the apparent difference between the brightness of ζ and λ *Pavonis* is greatest when the line joining the two stars makes an angle of 25° 23′ with the vertical.

A simple diagram may explain this far more directly and clearly than a literal or trigonometrical statement.

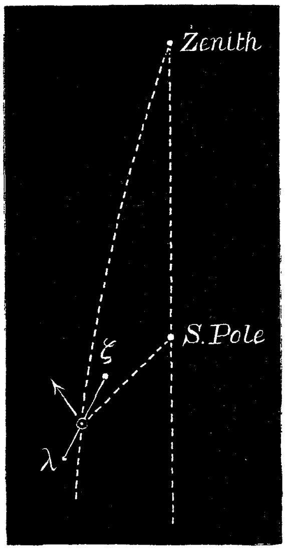

Fig. 1.

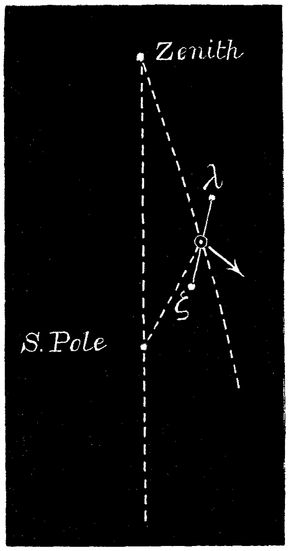

Fig. 2.

In Fig. 1 we have the position of λ with respect to ζ when the magnitude of λ with relation to ζ seems at a maximum. In Fig. 2 the magnitude of λ is at a minimum with relation to ζ *Pavonis*.

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[*] *Harvard Annals*, vol. xix. Part 2, p. 252.

1899MNRAS..59..524R

In the investigation already referred to at the beginning of this paper (*Monthly Notices*, vol. lvii. April 1897, p. 483) I obtained an angle of 32° with the vertical, as that at which the lower of two stars seemed to reach its maximum brightness.

In this investigation the angle arrived at is 25° 23′. The difference of $6\frac{1}{2}°$ is accounted for by the fact that however great the care taken to keep the position of the head erect, there is always a small inclination when looking through an ordinary theodolite, the instrument used at Lovedale. On the other hand, in naked-eye determinations there is no tendency to any inclination of the head.

Combining both results the following definite conclusions regarding the influence of relative position on apparent magnitude may be accepted:

(1) Through the unequal sensitiveness of the retina the lower of two stars will appear relatively brighter than it actually is.

(2) The maximum increase in relative brightness does not take place when one star is directly under the other, but *after* it has passed the vertical. The angle of maximum variation is about 30°.

(3) The amplitude of relative change due to this cause amounts to no less than three-fourths of a magnitude.

(4) The similarity between the results obtained when both eyes are used, and when only one is used, naturally points to the fact that the error is of the same magnitude and character whichever eye is used.

A simple experiment settled the locus of the error. I had rather inclined to the view that the *seeming* variation was more mental than physiological—that, in fact, the eye saw perfectly enough, but the brain behind, for some reason or another, put its own interpretation on the sensations presented to it.

When the constellation *Pavo* was low down, and when therefore λ seemed about $0^{m}\cdot 8$ brighter than ζ, I adopted an exceedingly undignified mode of observation—viz. looking at the constellation with head downwards, my two legs being the window through which the observations were made.

To my thus looking the positions of the stars were the same as they would be if viewed in orthodox fashion: *Pavo* did not seem upside down. The brain had received the picture presented to it, but from association or some other mental sense it turned the impression upside down, thus making the view appear more real.

There was this important difference however, λ *Pavonis was no longer* $0^{m}\cdot 8$ *brighter than* ζ; *it was instead over half a magnitude fainter.*

Indeed, all the magnitudes of the stars in the neighbourhood were what they would be after an interval of twelve hours. No more decisive proof could be forthcoming of the cause of

1899MNRAS..59..524R

the variation. It was entirely confined to the retina. Further, the phenomenon is in no way restricted to one or two observers. If this were so I would not enter so fully into the matter. Instead of being restricted, I believe it to be a condition of vision.

I have already referred to well-known observers, Barnard, Pickering, and Chandler, who have abundantly testified to its existence, as far as they were concerned. Recently Dr. W. J. S. Lockyer, in his valuable dissertation on the period of η *Aquilæ*, finds that Schmidt's observations are affected by a systematic error depending on the relative position of the variable and its comparison stars.

Indeed, there are few observers who have made the visual determination of star magnitudes a particular line of research who have not been brought face to face with the difficulty.

Lovedale: 1899 *May.*

Note on the Construction and Use of Réseaux.

By Arthur R. Hinks, M.A.

Very little has been published on the construction and use of *réseaux* since Professor Vogel described in the *Bulletin du Comité Permanent*, vol. i. p. 86, the experiments undertaken by Dr. Scheiner, when, at the inauguration of the Astrographic Chart, the study of the subject was confided to the Potsdam Observatory. A few scattered references show that astronomers engaged on the chart work have generally followed Dr. Scheiner in endeavouring to secure extreme fineness of the photographed image of the *réseau* by the use of very finely ruled *réseaux* placed very nearly in contact with the plate.

Some experiments which I made last year suggest that this procedure may with advantage be modified.

In the first place, it seems doubtful if one should aim at making the image of the *réseau* line as fine as a micrometer wire, unless Professor Turner's form of micrometer is used, in which the point of intersection of the line with a glass scale has to be *estimated.* It is certainly easier to set a wire upon a line well defined, but broader than itself.

Secondly, it is inconvenient to be obliged to place the plate very close behind the *réseau.* The use of commercial plates at any time is then quite out of the question, since they frequently deviate from flatness by several tenths of a millimetre. And even patent plate-glass plates occasionally have sufficient curvature to bring them into contact with and cause damage to the *réseau.*